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ABSTRACT

The following approaches to mathematics education, represented in German-speaking countries mainly, are analyzed: (1) the structuralist/formalistic approach; (2) the genetic approach; (3) the approach by elementarization; (4) the problem-solving approach; (5) the approach of Freudenthal; (6) the approach by project-orientation; (7) "anthroposophical" approaches; and (8) the approach by application. Similarities and differences are stressed, in the hopes of yielding some contribution to a theoretical foundation of mathematics education. It is felt that there is a current need for theoretical augmentation and guidelines and that this presentation highlights the norms and values of each analyzed approach. Suggestions are made for further study that could lead to a "meta-theory" of mathematics education. (MP)

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ON SOME TRENDS IN GERMAN MATHEMATICS EDUCATION

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Abstract

There is a need for a broader theoretical foundation of mathematics education (cf. BEGLE 1979 and BEGLE/GIBB in SHUMWAY 1980).

One step in this direction might be an analysis of different approaches to mathematics education with respect to their norms and values, which are incorporated in each of them at least implicitly.

Eight approaches to mathematics education, represented in German speaking countries mainly, are analyzed in the aforementioned way:

- the structuralistic/formalistic approach,
- the genetic approach
- the approach by elementarization,
- the problem solving approach,
- the approach of H. Freudenthal,
- the approach by project-orientation,
- "anthroposophical" approaches,
- the approach by applications.

Analogies and differences are outlined, which may be synthesized and incorporated into a broader (meta-) theoretical foundation of mathematics education.

1. Introduction

BEGLE 1979 deplores the absence of a "broad theoretical foundation" in mathematics education.

"An Agenda for Action" (NCTM 1980), oriented towards direct practical advice mainly, does not present such a theoretical foundation, too. There would be no need for such a foundation, of course, if the underlying "philosophy" in mathematics education is to be pragmatism (including questions like, "What is the trend of the market?"). In this way, slogans like "revolution in mathematics", "New Math", "back to basics" and "problem solving" (cf. KRULIK 1980) may appear and disappear. There still remains the additional need (except practical advice for today) for a theoretical framework, which "provides a basis for " a common "discussion" and for coordinated long term "research efforts" (cf. BEGLE 1979 and BEGLE/GIBB in SHUMWAY 1980). Mere pragmatism may result very often in eclecticism (cf. BUSSMANN/HEYMANN/LORENZ/SCHOLZ/SEEGER 1980), which had been deplored by BEGLE 1979, too. There is a need for theoretical augmentation and guidelines now!

A possible approach in this direction might be an analysis of different trends in mathematics education, e. g. like those, mentioned above.

In this paper, different trends in Western German mathematics education are analyzed. Similarities and differences are stressed, thus, yielding a possible contribution to a theoretical foundation of mathematics education.

To categorize this paper in the terminology used in SHUMWAY 1980, it may be labeled "philosophical research" (p. 24) or "meta-analysis" (p. 25,26).

2. Starting-point, objectives and procedure

The starting-point of this study is given by the following experiences (theses):

- There are different approaches in mathematics education. There is no distinction made between concepts as "approach" or "trend".
- Systems of values play an important part in mathematics education. Different approaches are very often determined by different norms or different emphasis placed on the objectives in mathematics education (e. g. rigor vs. creativity
- Very often, it is difficult in mathematics education (and other social sciences), to make a clear distinction between "facts" and "values". For example, "mathematical ability" is treated as a fact. But it is only defined with respect to a specific set of tests or problems (curriculum), which determine or include the educational objectives (i. e. values), too. This "theory-burdening" of all observations is a well-known theme, especially in the philosophical discussion (cf. BAUERSFELD in ATHEN/KUNLE 1977, EINSTEIN in V. WEIZSÄCKER 1977a, FEYERABEND 1970, KANT 1787, POPPER 1975).
- There is a lack of consciousness with respect to the "hidden" norms or values, incorporated in every approach. This statement could be derived upon from a broader review of the literature and practical suggestions in mathematics education (cf., e. g., BEGLE 1979).
- In some scientific disciplines related to mathematics education such as education, psychology, philosophy, sociology and science education, particularly in the German speaking countries, there is a strong movement

towards reflections and analysis of the specific subject with respect to philosophy of science (cf., e. g., BREZINKA 1978, KÖNIG 1975a, 1975b, 1978).
These statements require more confirmation and evidence than can be given here. This will be outlined in another paper.

Following objectives could be derived from the previous statements:

- Norms and values, underlying all kind of approaches, should be revealed and should be made conscious.
- Therefore, mathematics teachers and educators may gain the opportunity to determine and become conscious of their respective standpoint within the various approaches.
- Synthesizing different approaches may yield additional possibilities of innovations in mathematics education.

The procedure, of course, which leads from the starting point to the objectives by the analysis of different approaches, is based on specific (meta-) norms (or values), as well as the analyzed approaches themselves. For reasons of internal consistency, they are quoted here.
There is set value on:

- consciousness to norms and a need for their justification,
- plurality and openness in view of different value systems (cf. FISCHER 1979), as well as some kind of
- systematics, when describing and analyzing different approaches and the norms, on which they are based.

These leading ideas can be derived in some respect from the present discussion in the general philosophy of science. Today's tendencies of unifying the two main branches in the present philosophy of science, the analytical and historical-

hermeneutic, lead to the more informal, genetic semi-systematic procedure, carried out in this study. This had been derived upon mainly from the analysis of authors like KUHN 1976, 1978 and LAKATOS/MUSGRAVE 1974. The discussion between STEGMÜLLER 1973, 1979a, 1979b, 1980 and FEYERABEND 1977a, 1977b, 1978 can be taken as an example, which demonstrates the convergence of the two trends in the philosophy of science. This issue demands special and careful analysis which unfortunately, cannot be treated here because of space limitation.

3. Restriction of the area of analysis

The foregoing discussion should make it clear, that the term "mathematics education" should be used in a very broad sense (especially as a consequence of "plurality" and "openness"). With respect to the interdisciplinary character of mathematics education, different trends should be analyzed in related areas like mathematics, general education, psychology, philosophy, science education and other social sciences. This work could be supported particulary by authors like ADORNO 1979, BORN/OTTO 1978, BERNAYS 1976, BREZINKA 1978, BÜTTEMEYER/MÖLLER 1979, KLINE 1980, KÖNIG 1975a, 1975b, 1978, KUHN 1976, 1978, LAKATOS 1974, 1976, LORENZEN 1974a, FEYERABEND 1977a, 1977b, 1978, STEGMÜLLER 1973, 1979a, 1979b, 1980, WEYL 1966 (no complete list!). Additional analysis in this direction will be performed later on.

But even exclusive restriction to the area of mathematics education is still combined with problems of choice:
On one side, historical analyses should be carried out like those of BIDWELL/CLASON 1970 in Northern America and

HESTERMEYER 1969 and SCHMIDT 1978 in Western Germany. This track cannot be followed here.

On the other side, even the restriction to contemporary trends in mathematics education is still connected with problems of choice. For reasons of availability, trends in German speaking countries are mainly analyzed.

What can be regarded as a "trend" or an "approach" in (German) mathematics education, could be derived from the work of e. g. LENNÉ 1969, ATHEN/KUNLE 1977, BÜLTS 1978, VOLK 1979 and from the discussion about "Sense and Meaning of Mathematics Education" ("Sinn und Bedeutung der Mathematikdidaktik") in the "Zentralblatt für Didaktik der Mathematik" (ZDM) 1974.

Therefore, as a first approximation, a "contemporary trend in (German) mathematics education" is determined by a groupe of mathematics educators, carrying the same paradigm (in the sense of KUHN 1976), who strove for influence on the teaching and learning of mathematics (in the FRG mainly) within the last ten or fifteen years. Such group represents a trend, if all members are oriented towards one central idea at least. Often, a "background- theory" in the sense of VOLLRATH 1979 is an element of such trend.

Before presenting the list of trends to be analyzed, the following remarks should be taken into account:

a) This list can by no means, be complete.

b) The sequence in which the trends are treated here, will not be "well-ordered".

c) Many of these approaches have several similarities. The same objectives are emphasized only in different ways very often. Names, quoted in connection with a

specific trend, can frequently be applied to other trends as well. Therefore, this list cannot represent a partition of the area of (German) mathematics education.

d) The following analysis shall not be restricted to specific, "very promising" approaches. Otherwise, there would be the need for premature set-ups and justification of criterions of selection, which would be inconsistent with the abovementioned meta-norm "openness" with respect to different systems of values.

e) Attempts shall be made to describe the different approaches from the viewpoint of their representatives. Nevertheless, additional valuation should be carried out, too. But this procedure should be made clear respectively.

f) The same area of approaches to mathematics education could be subdivided from other points of view, as well.

g) It is not possible to present a complete analysis of any one trend. Sometimes, the whole approach is analyzed nearly exclusively by reference to one mathematics educator, who is considered a typical representative of that specific trend.

h) The treatment of trends can only present an impression of a situation(matter) at a given moment because changes are occurring rapidly. All approaches are interwoven and are evolving continuously.

The trends to be analyzed, are as follows:

- the structuralistic/formalistic approach,
- the genetic approach,
- the approach by elementarization,
- the problem solving approach,
- the approach of H. FREUDENTHAL,

- the approach by project orientation,
- "anthroposophical" approaches,
- the approach by application.

Each of these approaches is analyzed in its own section in the following way:

(i) The "hidden" norms of every approach should be revealed by analyzing its criterions of relevance and choice, their specific terminology and their methods of verification (cf. KÖNIG 1978).

(ii) Each section is closed by a summary of the specific norms, including their different emphasis.

4. The structuralistic/formalistic approach

This approach, well-known under the name "New Math", can be connected with people like DIEUDONNÉ, MESCHKOWSKI, H.-G. STEINER, PAPY, REVUZ and , to some extent, GRIESEL, too. This wave was dominating in the US in the sixties (cf. KRULIK 1980). It had its strongest influence upon mathematics education of the FRG with a five year delay, approxemately from the mid-sixties until the mid-seventies at least. Although the "back to basics" and (just now) the problem solving movement had started here, too, there is still a strong influence of the structuralist view of mathematics in education. This statement is especially applicable to France (the home of BOURBAKI, cf., e. g., GLAESER 1980) and many Eastern European states (cf., e. g., WEBER/WALSCH1975 from the GDR and SEMADENI 1976 from Poland, the home of TARSKI). It is not necessary to repeat the well-known discussion of the history and difficulties of this approach (cf., e. g., LENNÉ1969, KLINE 1974, BESUDEN 1975). The major goal of the subsequent analysis should be to

reveal the specific norms (in the manner , mentioned in 3.) as they could be observed in the FRG mainly.

The general objectives of this approach define the criterions of relevance and choice. These can be imbedded into a general "ideology of a mathematical- scientific culture", carried by the representatives of this approach (cf. LENNÉ 1969, p. 82). As far as mathematics is concerned, the frame is defined by the approach of BOURBAKI. Therefore, major emphasis is placed upon structures and logical and conceptual rigorness. Focusing on the three matrix-structures (order, topological and algebraic structures) results in an uniform, systematic and very economical description of the different areas of mathematics. Therefore, important additional norms of this approach are uniformity, systematics and economy (esp. with respect to proofs).

As mathematics is looked upon as the"science of abstract structures" (cf., e. g., V. WEIZSÄCKER 1971, BERNAYS 1976), the ability to formalize or to abstract is emphasized more by representatives of this approach than intuition and content-orientated thinking. Therefore MESCHKOWSKI 1973, p. 19, claims (thus defining educational objectives, too): "Working with mathematics will train objective thinking, protects from inadmissable generalization, it produces a more accurate language."

The structuralist view of mathematics very often defines the framework for (empirical) research, too. Therefore, the work of PIAGET is most interesting for representatives of this approach because he is a structuralist, too (cf. his "groupings" and, e. g., the work of DIENES/JEEVES 1968, 1972).

After outlining what is considered relevant, additional

remarks shall be given about aspects of teaching mathematics, which seem to be more or less irrelevant to representatives of this approach:

a) "There is a rejection of general educational objectives, extending beyond mathematics, and, last but not least, of general theories, which might lead to a restriction of mere subject matter-oriented demands from a mere encompassing point of view." (LENNÉ 1969, p. 82). In particular, affective or social educational objectives are seldom taken into consideration. The striving for performance and success is emphasized mainly in the non-specific mathematical area (LENNÉ 1969, p. 88).

b) Consequently, little emphasis is placed upon methods of teaching (cf. MESCHKOWSKI 1973, p. 20). Therefore, representatives of this approach care only little about teaching or learning processes. Such processes are of some interest to these persons as far as they can be modelized by structures similar to mathematical structures. Therefore , very often, they favor the theory of PIAGET, as it has been already mentioned above.

The methods of verification are influenced by the aformentioned leading ideas, quite essentially, too. Such methods are to be applied, if it is necessary to check whether statements like the following one can be accepted or not:"It is useful, to teach the mathematical subject matter A to the class B at the time C by the method D".

The mere analysis of the mathematical content and its internal logic with respect to the criterions of relevance, as mentioned above, is dominating. Many papers in the journals "Der Mathematikunterricht" (Mathematics Instruction)

and "Der mathematisch-naturwissenschaftliche Unterricht" (Mathematics and Science Instruction) had been oriented in this way for a long time. Empirical investigations in the area of this approach are carried out to the near exclusion in the U. S.. They are conducted only as far as the research design comes up to the standards of the aforementioned criterions "objectivity", "rigorness", "universal validity" (cf., e. g., SUPPES 1974, LESTER 1975).

The foregoing analysis can be summarized as follows:

More emphasis is placed upon

a) the subject mathematics, especially upon
b) structure-oriented mathematics
   (priority is upon description instead of development of mathematics),
c) the mathematical terminology,
d) precision and formal- logical deduction,
e) systematics,
f) objectivity,
g) striving for performance and success
   (priority is upon selection instead of encouragement),

less emphasis is placed upon

h) general educational objectives,
i) affective and social educational objectives,
j) methods of instruction,
k) the process (genesis) of understanding and mathematics.

For reasons of simplification and clarification, the analyses in the subsequent chapters refer to those in the previous ones.

5. The genetic approach

This approach still (resp. again) has a very strong influence on German mathematics education. Well known representatives are, e. g., WAGENSCHEIN, WINTER and E. WITTMANN. This school has a long tradition and has been described in the recent work of SCHUBRING 1978 very carefully. The following analysis refers mainly to his book.

Looking for the criterions of relevance and choice, leading this apprcach as central ideas, results to the cues "development" and "process". These can be specified as follows:

Representatives of this approach feel that it is very important for the subject matter (mathematics), as well as for the learning student to take into account their previous and future development. "Previous" refers to mathematics, "future" to the students, mainly.

Selecting and preparing material on the "macro-level" historical analyses of developmental process within mathematics should be included (this method is called "historical-genetic").

Representatives of this approach favor the psychological-genetic method on the "micro-level" (i. e., classroom-teaching), which stresses the development of activities, which are to be anchored in every-day knowledge (cf. SCHUBRING 1978, p. 187/194).

Both methods adopt the "bio-genetic law", claiming parallelism between phylogenesis and ontogenesis.

The two methods should incorporate "the social mediation between learning individuum and subject to be learned" additionally (SCHUBRING 1978, p. 195).

For reasons of clarification, this approach should be

delimitated from the aforementioned structuralistic approach as follows:

a) The subject mathematics, esp. taking it as the science of structures, does not play such an important part within this area. Its representatives tend to a mediation between the subject matter (mathematics) and the educational-psychological aspects. They consider the epistemological process to cover both areas.

b) Consequently, methodological inquiries are of greater improtance here than in the structuralistic approach. It is particularly important to mention that "integral" teaching methods (including emphasis of, e. g., material analysis and restructuring-processes) are derived often from gestalt-psychology (cf. WITTOCH 1973).

c) Within mathematics, there should be more emphasis placedon the content than on the (logical) form (cf. SCHUBRING 1978, p. 1). There is more stress placed upon vague intuitive generation of ideas and conjectures, than formal-logical deduction.

d) Consequently, representatives of this approach prefer to select subject matter and problems, which correspond to a "dynamic" conception of mathematics. Mathematics is taken as an area fo special activities.Therefore, there is a re-discovery of geometry in contrary to algebra and more stress upon open-ending than closed problems (cf., e. g., MÜLLER/WITTMANN 1978).

e) Performance is expected within this approach,too, of course. But this expectation is not as intense as that within the formalistic/structuralistic approach. There is more demand on "creativity' and the "ability

to argue" (on a respective adequate level, cf. WITTMANN 1975, WINTER 1972) than on perfection and "absolute rigor".

f) Additional general educational objectives (affective or social ones) are not pursued within this approach.

This section shall be closed with some remarks on the terminology and the methods of verification . Concepts such as "mathematical abilities", are to be understood in the light of the aforementioned leading ideas (cf. esp. e)). Of course, very often, the area of meanings of terms, used by representatives of this approach, is broader than that in the formalistic/structuralistic one because there is less orientation towards the subject of mathematics. Methods of verifying statements of mathematics education are frequently "softer" than those applied within the aforementioned approach, stressing more relevance and content than significancy of rigor and standardized forms. Therefore, clinical studies are very often prefered in this area (cf. SHUMWAY 1980, too).

This section is closed by the following summary:
More emphasis is placed upon

a) the process of cognition (in connection with the subject mathematics, education, and psychology),
b) the "dynamical" aspect of mathematics (mathematics as an area of special activities),
c) argumentation oriented towards content and intuition),

d) the spontaneous generation of ideas and their local development (treatment of episodes rather than systematics in the mathematics lesson),

e) performance as creativity and an ability to argue,

f) methods of instruction like "discovery learning" or "learning by doing",

less emphasis is placed upon

g) formal-logical-deduction and mathematical terminology (cf. c)),

h) internal systematics due to the subject matter (cf. d)),

i) general educational objectives.

6. Elementarization of mathematics

This approach can be looked upon as a typical approach in German high-school (Gymnasien) mathematics education. Representatives are, e. g., BECKER 1974 and KIRSCH in ATHEN/KUNLE 1977 as well as many teacher-trainers for the German Gymnasien.

There are some analogies between this approach and the structuralistic/formalistic one. Especially in both trends more emphasis is placed upon the subject matter. Parallels and differences compared with the two approaches are outlined by referring to the typical paper of KIRSCH 1977 "Aspects of Simplification in Mathematics Teaching", presented at the third ICME in Karlsruhe 1976. While presenting the most essential goals, the main criterions of relevance and choice are given implicitely, too.

Structure-mathematics is still important to this approach, though with less rigerosity than that of the structuralistic/formalistic approach: "After basic clearifications,esp. in the works of H.-G. STEINER and WÄSCHE, and a period of perfectionistic realizations in the mathematics lesson ... one is just finding the right proportion ...." (KIRSCH 1977, p. 153).

One of the main objectives of this approach is to prepare mathematics for instruction by "simplification", to make "mathematical essence" better accessible.

Some connections to the structuralistic/formalistic approach and demarcation to more genetic or application-oriented instructions could be clarified by taking into account the position of PRINGSHEIM. KIRSCH 1977 says, that he had the "conviction, that mathematics is simplest and most fundamental, when it is mature and when it is taken in its most narrow sense and when all genetic elements and references to reality are strayed. Means to faciliate the access are supposed to be working out the conceptual core, generalization and abstraction as well as emphasis on the fundamental structures."

On the other side, KIRSCH (and other representatives of this approach) suggests to proceed from the pupil's existing pre-knowledge about mathematics to deeper understanding. This strategy is opposed to exaggerated tendencies of axiomatization, giving anologies to the gentic approach in this way. This impression is confirmed by KIRSCH'S recommendations, to "change the medium of representation" (cf. BRUNER) or to use "illuminating examples" (cf. WAGENSCHEIN'S exemplary principle!) to attain a better under-

standing of mathematics.

Though representatives of this approach take the selection and justification of curriculum material for mathematical instruction to be the main objective (cf. KIRSCH 1977, p. 151),thus, placing major emphasis on the material aspect, they pay strong attention to methodological inquiries, too. Insofar, they have a mediative position between the two aforementioned approaches.

There is no special emphasis placed on general educational and social objectives, too. This can be seen by the following statement of BAURMANN 1979, p. 258 (excerpt of a paper, presented at the annual meeting of the organisation of mathematics and science high-school teachers (MNU) in Hannover 1979, main theme: elementarization):
"First, the didactics of a subject (mathematics education) may not be confused with its methods of instruction. Second, it is not a handmaid of the general theory of education, it is not a social science, a social history of science, nor a theory of communication ... . The didactics of a subject deals with the "what" of the subject, with the way it is seen by itself, with the teaching and the learning of a discipline, beginning at its contents ... . The subject is the basis, its didactics are the exponent of a power."

The terminology, of course, is influenced strongly by the domination of the subject, but it is not as restricted and technical as in the structuralistic/formalistic approach. As methods of verification empirical investigations play a less important part than the "material-analysis", because

the subject (material-aspect) is dominating. Another important criterion is the "managability" with respect to the specific boundary conditions, set by the concrete situation (class).

The summary is as follows:

More emphasis is placed upon

a) the subject mathematics (defined by the the "ideal case" mathematics, taught at universities),

b) preparation of mathematics (structure-mathematics) by simplification (by use of methodological aids like those ones, used in the genetic approach),

c) systematics (due to the subject) combined with practical management, formed by the school-system,

d) methods of instruction like "questioning-developing" instruction or "frontal" instruction,

less emphasis is placed upon

e) objectivity and striving for performance (it depends very strongly on the special school-situation),

f) general educational objectives.

7. Problem solving

This approach is extremely well-known in the U.S. for a long time. Here "problem solving is the focus of school mathematics in the 1980s." ("An Agenda for Action", NCTM 1980). A good summary of the present discussion in

this country is given, e. g., in KRULIK/REYS 1980 and GOLDIN/MC CLINTOCK 1980.

In Western Europe, esp. in the FRG, this approach is still at its very beginning.

The main influence is coming here from the work of POLYA, too. The criterions of relevance and choice can be derived, as follows:

The primary intention of this approach is "training of thinking" (cf. POLYA 1967). This goal should be reached by concentrating on the "dynamical" aspect of mathematics (" mathematics in the process of development", "invention of mathematics", "reasonable guessing", cf. POLYA 1967). Major emphasis is placed upon the process of cognition by doing mathematics. Insofar, there is a strong semilarity to the genetic approach.

The following suppositions seem to be fundamental to this approach:

(1) Solving mathematical problems is one of the main objectives in mathematics education (cf. NCTM 1980).

(2) Rules and patterns of cognition processes and problem solving processes may be discovered.

(3) Problem solving processes, sometimes problem-specific, called "heuristics", "strategies" or "heuristic strategies", may be classified (into, e. g., trial-and-error, generalization, specialization etc.).

(4) As such strategies are supposed to be important for doing "good mathematics", it is useful to place them into a list of educational objectives (cf., e. g., WITTMANN 1975).

(5) It is possible and, therefore, useful, to learn and

teach such strategies.

(6) Additional analysis of poor problem solving and mistakes is useful (cf., e. g., KILPATRICK 1967, KANTOWSKI 1974, RADATZ 1979, ZIMMERMANN 1977).

"Static" mathematics (results of mathematics) are not as emphasized as in the structuralistic/formalistic approach. General educational and social objectives are not pursued either.

The terminology in this area is similar to that of the genetic approach. Although, the domain of meanings of concepts is somewhat more well-defined with less room for ambiguity. This depends on the fact, that in this area a great amount of empirical research (used as methods of verification, too) is conducted, which demands a more detailed operationalization of the terminology.

The results of this analysis can be summarized as follows:
More emphasis is placed upon

a) solving mathematical problems ("learning of thinking", connection of mathematics, cognitive psychology and epistemology),
b) the dynamical aspect of mathematics,
c) the systematical generation of ideas by applying heuristic strategies,
d) performance as problem solving performance,
e) empirical investigations of teaching and learning of problem solving,
f) discovery learning,

less emphasis is placed upon

g) imparting mere material,

h) formal-logical deduction,

i) objectivity,

j) educational objectives.

8. FREUDENTHAL'S approach

The approach of FREUDENTHAL is presented in his 1974 and 1978 mainly (cf. VOLK 1979, too). At least two aspects or two levels must be distinguished:

On one side, there is his main concept for a "Science of Mathematical Education" (FREUDENTHAL 1978, Ch. 4), on the other side there is the preparation for this concept including analyses in the area of philosophy of science (Ch. 1-3).

A few remarks will be presented now on this level of preparation.

FREUDENTHAL is one of the very few mathematics educators (cf., e. g., BIGALKE 1974, too), who is making a conscious effort to incorporate his educational conception on a higher level into a general background philosophy about science, education and science of education. To some extent, this educational conception is acting on itself in this way, thus satisfying the criterion of consistency, which is one of his criterions for scientific work. Awful examples in the praxis of educational research are used for reasons of demarcation.

In analyzing his philosophical background, there seem to be many parallels to the epistemological anarchism of FEYERABEND 1970:

a) Both favor the use of "ad-hoc-methods" (resp. theories) in solving real problems against logical deductions from general theories. Both refer to the history of mathematics and physics to support their statement (cf. FREUDENTHAL 1978, p. 10/11, FEYERABEND 1977a, Ch. 8).

b) Using rational analysis, FREUDENTHAL (1978, Ch. 1, Ch. 2) comes to the conclusion, that elements of faith (hence, elements of irrationalism) are necessary and inevitable in education and to a lesser degree in science. This opinion is supported by the findings of SUYDAM/OSBORNE 1977:"They argue that conflict of different values and beliefs is the most critical problem in changing educational practice." (FEY 1979, p. 500). This point is stressed even more by FEYERABEND 1977a, p. 392.

In this manner, FREUDENTHAL is revealing and justifying the "norms " of his approach. This procedure seems to occur seldomly in mathematics education. In this way, the base of argumentation can be seen very clearly by a possible "applicator".

He presented his main ideas with additional concrete examples in a lecture on "Major Problems of Mathematics Education" during the 4th International Congress on Mathematical Education in Berkeley 1980, evoking the image of HILBERT'S 1900 address at the ICM in this way (cf. FREUDENTHAL 1981).

This approach has some semilarities to the genetic and problem solving approach, as the learning process is of major interest (cf. FREUDENTHAL 1978, p. 167). Learning processes should be teaching processes as well, because of

the respective interaction between pupils and teachers in the classroom. Insofar, there should be a distinction made between "learning processes" in the sense of (mathematics) educators and "learning processes" in the sense of psychologists (FREUDENTHAL 1974, p. 124).

Especially, the observation of discontinuities within such learning processes (aha-experiences of gestalt-psychology) is considered to be essential. These phenomenons can be recognized "in the individual only, not in the average. Discontinuities are over-looked when formulating an average." (FREUDENTHAL in VOLK 1979, p. 187). Consequently, FREUDENTHAL prefers clinical methods to analyze learning processes.

There is a stepwise development of learning processes especially within mathematics, whereby activities within a learning process on one level may be the objects of activities of a learning process on a higher level (FREUDENTHAL in VOLK 1979). This fact has to be taken into account when teaching mathematics. Furthermore, FREUDENTHAL takes the view, like many representatives of the genetic approach, that learning could be taught optimally, by use of "paradigms" ("good examples", cf. WAGENSCHEIN 1962). Moreover, mathematizing activities should be stimulated; more emphasis should be placed upon "directed re-discovery" (SOCRATES' method) than upon teaching accomplished mathematics (cf. the remarks on POLYA in 7.). Hence there are some semilarities to the problem solving approach. In addition FREUDENTHAL prefers "local ordering" on a respective adequate level of rigor to an "absolute" rigor or "global-systematic" procedure (cf. FREUDENTHAL 1973, p. 142).

Therefore, he makes - inspite of many parallels- a clear demarcation-line to the genetic approach:"... corresponding to his opinion, education, using the genetic method, is too straightforward, too much stated by the teacher and will not leave enough free room for mathematization to come out of the original activities of the pupils." (SCHUBRING 1978, p. 171).

Thus, FREUDENTHAL'S engagement for mathematics is combined with an engagement for the pupil, linking up both with a liberal process of cognition.In this way, he excludes the teaching of heuristics, listed up in a fixed catalogue and giving a demarcation to the problem solving approach,too.

No further educational (or sociological) objectives are presented by FREUDENTHALexplicitly. Sofar, I have covered criterions for relevance and choice.

Now, coming to his terminology, it seems to be somewhat original. Nevertheless, it seems much clearer than that of other approaches, which is mainly attributed to the careful reflection about it (cf. FREUDENTHAL 1978, p. 183 "Language as a Vehicle of Research"). Striving for enrichment of concepts, he tends to a very cautious specification, differentiation and systematization as well, up to first attempts of an didactical phenomology (cf. FREUDENTHAL 1978, p.292).

Checking statements about subject matter, he prefers a logical-material analysis as a method of verification. Checking empirical statements (e. g., Learning processes are working in the following way:...) he prefers clinical methods (see above). He does not exclude the possibility, that the separation of the learning process into different

levels is "procuced by the structure of mathematics itself." (FREUDENTHAL in VOLK 1979, p. 189). Epistemological inquiries might be pursued looking into "the" nature and the possible respective interaction of mathematics and learning processes. Cf., e. g., the platonistic epistemology of V. WEIZSÄCKER 1977 and the evolutionistic one of LORENZ 1973. At this point, the question has to be posed, whether one can be certain of one's own learning (of mathematics) to satisfy laws of the same security, which explain the structure of the interior of the sun (cf. the theory of EDDINGTON in SCHWARZSCHILD 1965), thus giving sense of their application to "the interior" of the pupil in a similar way (cf. FREUDENTHAL 1974, p. 123).

The chapter is closed by the following summary:

More emphasis is placed upon

a) learning processes and their discontinuities,
b) the dynamical aspect of mathematics,
c) "local ordering" on an adequate level of rigor,
d) "ad-hoc" methods in problem solving,
e) elements of faith,
f) a large area of freedom when doing mathematics,
g) the method of "directed re-discovery", (SOCRATES' method),

less emphasis is placed upon

h) absolute rigor (cf. c)),
i) global systematics (cf. c), d)),
j) objectivity (cf. e)),
k) the performance principle,

1) general educational objectives.

9. The approach by project-orientation

This approach can be taken as an outcome of the student-movements at the universities during the end-sixties in the FRG.

The main cue is "social relevance" of the subject to be taught, which should be attained by reality-oriented projects adjusted to the needs of the pupils as opposed to those of mathematics or mathematicians, normally encompassing more than one subject.

A representative description of this approach is given by MÜNZINGER 1977. Possible reasons for the instruction of mathematics in this manner are, e. g., given by DAMEROW and BÖLTS in VOLK 1979, p. 100/285 and BÖLTS 1978.

One has to take into account, that this approach had been developed as a consequence of experiences at the "Gesamt-schule" (an integration of the German "Hauptschule" (9-year elementary school), "Realschule" (middle-school) and"Gymnasium" (high-school)) in the Federal Country of Hessen.

BÖLTS (in VOLK 1979, p. 285) quotes the following dis-advantages of the classical mathematics-curriculum in normal class-room situations:

- there is a very asymmetric, standardized structure of communication ("frontal instruction"),
- pupils have to do single and receptive work,
- pupils cannot plan or critisize their lessons meaningfully,
- the teacher imparts a special body of knowledge (of the subject matter) to the pupils, which he cannot legitimize.

Such deficits should be overcome by project-orien tated mathematics instruction, which is characterized by MÜNZINGER in VOLK 1979, p. 214, as follows:

- reference to reality,
- integration of different subjects,
- social relevance (see above !),
- reference to the needs of the pupils,
- collective realisation (cooperation, no presure of competition),
- self-evaluation.

Hereon, are given the criterions of relevance and choice for this approach.

They are presented on a "meta-level" by a background philosophy of this approach, which had been developed by BÖLTS 1978. This philosophy already had an impact on the aforementioned negative experiences (ideology-burdening of all observation, cf. 2.). The main ideas of BÖLTS' 1978 are outlined here:

The starting-point of this study is given in the thesis, that all approaches in mathematics education are oriented too much towards the subject of mathematics and too little towards the "concrete" pupil and his position within society. Consequently, BÖLTS (1978, p. 31) assumes, that "didactics of a subject is a social science primarily.". Therefore, more emphasis should be placed upon furtherance than on selection. There is greater expectation of performance in the social area (social-integrative behavior, ability for self determination) than in the area of the subject (mathematics).

BÖLTS 1978, p. 39/40, presents the general leading ideas

of this approach in the form of deficit hypotheses (abbreviated version):

a) Theory and justification-deficit

There is a lack of a comprehensive theory, in which "one can describe and analyze the social existence of the pupil" in all its dimensions. The establishment of mathematics education does not justify its activities sufficiently (in the FRG, but, cf. BEGLE 1979, too!).

b) Socialisation-theory-deficit

Conditions and effects of socialisation for learning mathematics had been neglected too much until now.

c) There is a lack of evaluation- and control-instruments in the area of mathematics education (in the FRG!).

d) Praxis-deficit of theory oriented didactics of a subject (i. e., mathematics education).

e) Self-reflection-deficit of mathematics education.

Many of these statements may be turned into leading ideas and are remarkable. But there are difficulties in at least two areas:

(i) The methods, used to prove these deficits, are very problematical (cf. the criticism of VOLLRATH 1978 !),

(ii) Furthermore, BÖLTS analysis suffers from many of those deficits, with which he reproaches other mathematics educators and which he claims to overcome:

He does not present a comprehensive theory (cf. a)). With regard to that, many of his conceptual constructs are too vague and not carried out enough. Thus, the "concrete pupil" is rather non-concrete and distant from praxis (cf. d)). What are the "economical, social and political interests of the pupil"? Where will the answer to this question come from ? Does the pupil already know (all) his interests ?

If he should know them, would he not be overtaxed in a similar way, as if he would be asked to learn too much of (pure) mathematics ? Should it not be one task of education too, to form or to evoke such interests ? The methods, applied by BÖLTS in analyzing different approaches to mathematics education, do not remove the lack of instruments of evaluation (cf. c)).

Emphasizing the leading ideas of social learning, encouragement, revealing the economical dependence and of the reduced importance of the subject matter, BÖLTS is carrying "ideology" as well as those he is critisizing. Generally, this cannot be avoided (cf. FEYERABEND 1970). But it seems to be not reasonable to call an ideology a "social wrong consciousness" (as BÖLTS did), for the judgement of "right" or "wrong" demands an ideology again, i. e. a system of leading ideas or basic values, which should be made explicit to avoid unsuitable dogmatism . Insofar, BÖLTS' approach suffers from a lack of "self-reflection", at least, as well as those he critisized (FREUDENTHAL'S approach can be taken as an "counterexample" to this and other theses).

In this way, the criterions of relevance and choice had been outlined.

The "sociological" terminology is determined by the fact, that BÖLTS looks upon mathematics education as a part of the social sciences. There is , of course, a reverse influence, too.

A similar statement can hold with respect to the methods of verification, not only on the "meta-level" (cf, e. g., BÖLTS 1978, p. 41), but also on the "object-level" (cf. MÜNZINGER 1977, p. 209).

The foregoing discussion may be summarized as follows:

More emphasis is placed upon

a) the "concrete" pupil and his needs,

b) social learning, a collectivistic picture of man,

c) encouragement (and equal chances for all) instead of selection,

d) reference to applicability and use of mathematics (mathematics as a tool in every-day life),

e) teamwork and project-teaching,

less emphasis is placed upon

f) mathematics as a science,

g) mathematical terminology,

h) rigor, systematics, objectivity,

i)performance principle (cf. c)).

10. "Anthroposophical" approaches

Approaches are to be discussed here, as they are suggested by SCHUBERTH 1971 within the general theory of education of R. STEINER 1972 (cf. VOLK 1979, p. 350, BORN/OTTO 1978, p. 250) or by HEFENDEHL 1978 within the frame of the "non-directive psychotherapy"of C. ROGERS 1972,1976.

These trends have more similarities to the aforementioned one than to the others: in both cases the pupil is in the centre of discussion, the subject is of secondary importance. Approaches of this kind, put more emphasis on encouragement, too, than on selection (cf., e. g., SCHUBERTH in BORN/OTTO 1978, p. 260).

But while project-oriented instruction is oriented towards

a collectivistic view of man, the anthroposophists place more stress on man as an individuum . So SCHUBERTH (in BORN/OTTO 1978, p. 259/260) as well as ROGERS 1976, p. 171, quote both the deserving promotion "autonomical personality" ("to be one's own self, which one is in truth", ROGERS 1976, p. 173).

The following remarks will be concentrated on the approach of SCHUBERTH only, because there is a lack of information of the others in the area.

Though SCHUBERTH seems to put more emphasis on the subject matter (as done by representatives of the genetic or problem solving approach), the selection and development is determined by the underlying picture of man. Within this anthroposophical theory, the development of senses is a major interest. Geometry and the potential to use it to teach motoric skills, is of special importance to this end. Emphasis is put on application of mathematics, which are related to real-life situations (cf. VOLK 1979, p. 350).

Those statements and activities, which are taken to be relevant or to be chosen for the classroom, are not primarily determined by the subject, elements of psychological or epistemological theories (as it happens in all mentioned approaches except the project-oriented one), but by an integral individualistic picture of man. This conception seems to be elaborated much more by R. STEINER than the collectivistic picture seems to be worked out by BÖLTS, DAMEROW and MÜNZINGER.

The terminology and the methods of verification are influenced by the underlying main ideas and vice versa (as it had been in the aforementioned approaches).

The summary is as follows:

More emphasis is placed upon

a) the pupil as a whole personality,

b) general educational objectives (individualistic picture of man),

c) encouragement instead of selection,

d) methods like teaching in periods, self-experience, non-directive leading of conversations,

less emphasis is placed upon

e) mathematics as a subject, selection of material is oriented towards the picture of man

f) formal-logical deduction and precision, mathematical terminology, systematics and objectivity,

g) the performance principle.

11. The approach by applications

This approach is a renaissance in that time, for it can be seen in part as a reaction to the experiences of "New Math". Increasing economical presure (worldwide) urges mathematicians at universities (who are often teacher-educators, too) to demonstrate to the publicity (tax-payers) for which reasons they are payed and , therefore, orient themselves more to applications (cf. FUCHS in VOLK 1979, p. 228). The increasing importance of computerscience has a strong influence on this development, too (cf. several articles in the "Mathematical Intelligencer", e. g. that one of FRAUENTHAL in 1980, Vol. 3, no. 1).

The trend towards applications seems to be very strong in

the U. S., too (cf., e. g., the NCTM Yearbook 1979 on "Applications in School Mathematics").

In German-speaking countries ideas about applications are presented, e. g., by G. BECKER, BURSCHEID , BLUM, ENGEL, DÖRFLER/FISCHER, to quote a few names.

The major criterion (of relevance, too) in chosing material for teaching and learning is reference to reality. Insofar there is a connection to the project-oriented approach, but its representatives need not have such an underlying collectivistic picture of man (cf. 9.).

Hence reference to reality may be striven towards because of quite different reasons (cf. FUCHS in VOLK 1979, p. 330):

(1) it may be a mean to demonstrate the use of mathematics,

(2) it is very often a tradition (in German mathematics lessons, at least), to practise some recently acquired knowledge or techniques by solving applied problems,

(3) very often the teacher tries to motivate the pupil by beginning a new topic with an interesting real-life problem. In this way, he hopes later to introduce more difficult and abstract matters in a more effective way.

(4) The solution of problems of social relevance might be the focus of the mathematics lessons (as it is the goal of the project-oriented approach, too).

Consequently, teaching applied mathematics may be oriented towards the subject matter (as, e. g., in the elementarization approach) or towards the pupil (as in the project-oriented approach).

Corresponding to this very broad area of possible intentions, the terminolgy and the methods of verification cannot be described in an unique way.

Therefore the summary is very rough, too:

More emphasis is placed upon

a) reference to reality and the utility of mathematics,

b) encouragement and motivation instead of selection,

some emphasis is placed upon

c) "static" or "dynamic" mathematics,

d) different methods of teaching (broad range of methods),

less emphasis is placed upon

e) general educational objectives,

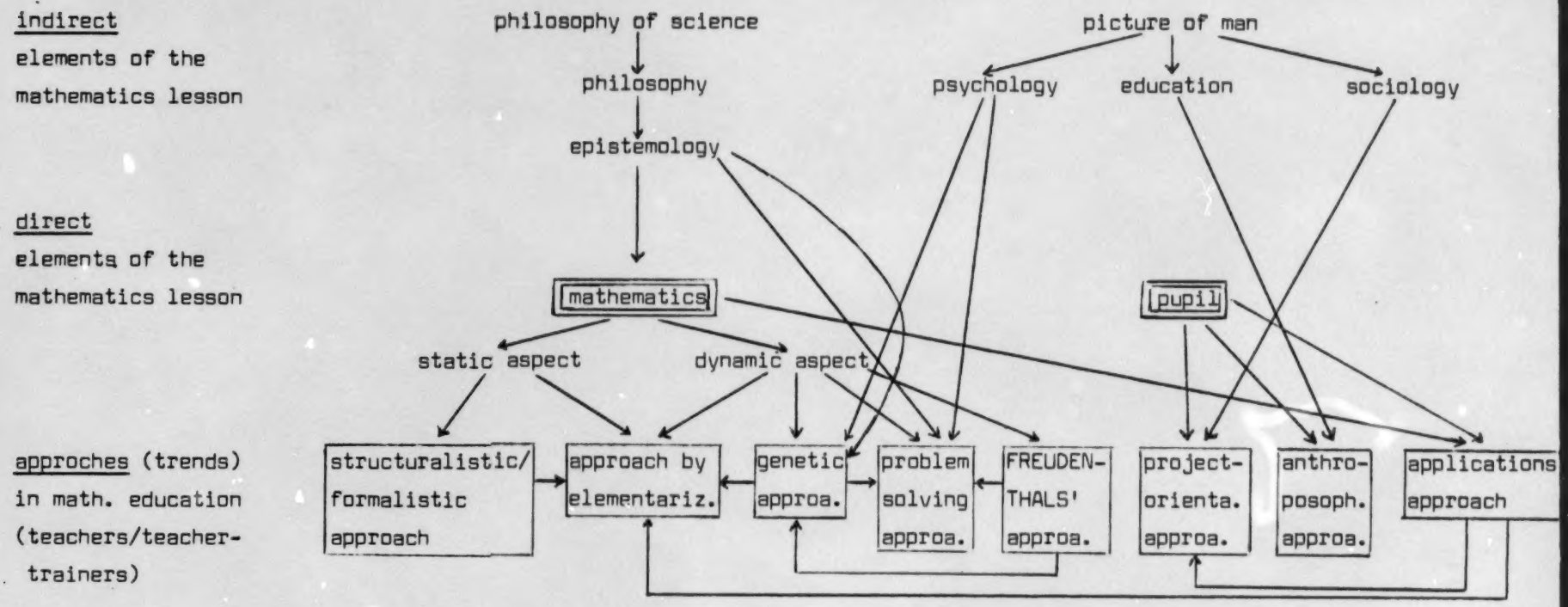
f) formal-logical deduction, systematics, objectivity, mathematical terminology.

12. Summary

An analysis of some trends in Western German mathematics education was presented. In the following chart shall be pointed out by arrows only the main interdependences of the approaches, which had been analyzed, including dependences on direct or indirect elements of mathematics education as well as their interdependences. Thus, if an arrow is not drawn, it does not mean, that there is no connection!

The concepts are arranged from the top to the bottom by increasing narrowness of their meaning. No more arrows have been drawn because of reasons of clearness.

Fig. 1



Connection of some trends in German mathematics education

Fig. 1

13. Final Remarks

Many mathematics educators deplore the state of eclecticism and the lack of standards in mathematics education (BEGLE 1979, BUSSMANN/HEYMANN/LORENZ/REISS/SCHOLZ/SEEGER 1980, cf. esp. the remarks of KAPADIA 1980 on the 4th ICME in Berkeley, they are similar to those of BEGLE 1979, p. 156, about the foregoing congresses).

Analysis of the "needs of the market", advice and orientation for the practical work is necessary (cf. OSBORNE/KASTEN 1980, NCTM 1980). However, it will not be sufficient for long term orientation and planning. Therefore, there is a strong need for a more theoretical frame, "to provide a basis for our discussions" (BEGLE 1979, p. XIX, and BEGLE/GIBB in SHUMWAY 1980, p. 9; cf. BAUERSFELD in ATHEN/KUNLE 1977, too).

Perhaps this "meta-analysis" (cf. JOHNSON in SHUMWAY 1980) of various approaches in mathematics education may yield some first rough "brick-material", which might lead to a way to "facilitate the edifices of broad theoretical foundations for the teaching and learning of mathematics, as well as, curricula content and organization." (BEGLE/GIBB in SHUMWAY 1980, p. 9).

The way to such "meta-theory" of mathematics education is, of course, still very long.

Some inquiries are presented at last, which seem to be necessary to come to further intermediate stations:

a) The analysis of the aforementioned approaches has to be carried out in depth.

b) The list of trends, which have been treated, must be completed. The different approaches in various countries (including different needs and norms) are to be enclosed.

c) The claim for revealing and justifying norms (cf. 2.) makes it necessary, to do just the same on a meta-level when constructing the framework for the analysis of different approaches. This had been only intimated until now and should be carried out.

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